Solution by G. B. M. ZERR, A. M., Ph.D., The Temple College, Philadelphia, Pa., and J. E. SANDERS, Hackney, Ohio.

$$\frac{\$A+\$B}{1+p}=\frac{\$6300}{1.05}=\$6000=\text{cost of both farms.}$$

$$\$A+\$B-\frac{\$A+\$B}{1+p}=\frac{\$p(A+B)}{1+p}=\frac{\$6300\times .05}{1.05}=\$300=\text{gain.}$$

$$C+\frac{p(A+B)}{1+p}=\frac{C+p(A+B+C)}{1+p}=\$4000+\$300=\$4300=\text{cost of dearer farm.}$$

$$\frac{A+B}{1+p}-\frac{C+p(A+B+C)}{1+p}=\frac{(A+B)(1-p)}{1+p}-C=\$6000-\$4300=\$1700$$

$=$cost of cheaper farm.

Also solved in a similar manner and with same result by *G. W. GREENWOOD*.

ALGEBRA.

171. Proposed by IDA M. SCHOTTENFELTZ, A. M., New York, N. Y.

$ay^2+a=bxy+cx$, $bx^2+b=axy+cy$. Solve for x and y.

Solution by G. B. M. ZERR, A. M., Ph. D., Professor of Chemistry and Physics, The Temple College, Philadelphia, Pa.

$ay^2+a=bxy+cx....(1)$. $bx^2+b=axy+cy....(2)$.

From (1), $x=a(y^2+1)/(by+c)....(3)$.

(3) in (2) gives $[(a^2+b^2)y^2+2bcy+a^2+c^2](cy-b)=0$.

$$\therefore\ y=b/c,\ y=-\frac{1}{a^2+b^2}\{bc\mp a\sqrt{[-(a^2+b^2+c^2)]}\}.$$

$$x=a/c,\ x=-\frac{1}{a^2+b^2}\{ac\pm b\sqrt{[-(a^2+b^2+c^2)]}\}.$$

Also solved by *MARCUS BAKER*.

GEOMETRY.

103. Proposed by PROFESSOR BEYENS.

Si le rapport du segment d'une base de la sphère à l'hemisphère est m/n, le rapport de l'hauteur du segment à deux bases qui resultera au rayon est égal à $2\sin\frac{1}{3}[\sin^{-1}(n-m)/n]$. [Problem 9699, *Educational Times.*]

Solution by J. R. HITT, Goss, Miss.; G. B. M. ZERR, A. M., Ph. D., The Temple College, Philadelphia, Penn., and G. W. GREENWOOD, B. A., Professor of Mathematics and Astronomy, McKendree College, Lebanon, Ill.

Let R denote radius of sphere, h the altitude of segment of two bases, $R-h$=altitude of segment of one base. Then, $\pi(R-h)^2[R-\frac{1}{3}(R-h)]/\frac{2}{3}\pi R^3$

$=3(R-h)^2(2R+h)/6R^3=m/n$. Therefore, $(h/R)^3-3(h/R)+2=2m/n$; or, $(h/R)^3-3(h/R)+2(n-m)/n=0$. Applying the proper trigonometric formula, $h/R=2\sqrt{(p/3)}\sin\frac{1}{3}\theta$, where $p=3$, $\theta=\sin^{-1}3q/2p\sqrt{(3/p)}=\sin^{-1}(n-m)/n$, $q=2(n-m)/n$. Hence, $h/R=2\sin\frac{1}{3}[\sin^{-1}(n-m)/n]$.

194. Proposed by **MARCUS BAKER**, U. S. Geological Survey, Washington, D. C.

Glass paper weights, having the form of a regular tetrahedron, are to be packed for shipment, each in a paper box. Wanted to know the size and shape of the smallest box for the purpose. How much empty space in each box?

Solution by the PROPOSER.

Shape of box = cube.

Edge of box $=s\sqrt{\frac{1}{2}}$, where s=edge of tetrahedron.

Empty space $=\frac{1}{3}s^3\sqrt{\frac{1}{2}}$.

Occupied space $=\frac{1}{6}s^3\sqrt{\frac{1}{2}}$=volume of tetrahedron.

Total space $=\frac{1}{2}s^3\sqrt{\frac{1}{2}}$=volume of box.

Of the twelve diagonals in the six faces of the box, the six edges of the tetrahedron coincide with one in each face.

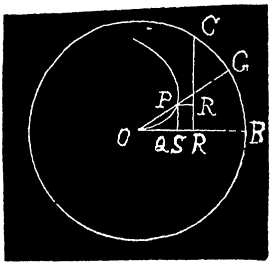
CALCULUS.

160. Proposed by **B. F. FINKEL**, A.M., M.Sc., Professor of Mathematics and Physics, Drury College, Springfield, Mo.

A dog at the vertex of a right conical hill pursues a fox at the foot of the hill. How far will the dog run to catch the fox, if the dog runs directly towards the fox at all times, and the fox is continually running around the hill at its foot, the velocity of the dog being 6 feet per second, the velocity of the fox being 5 feet per second, the hill being 100 feet high and 200 feet in diameter at the base?

Solution by **G. B. M. ZERR**, A. M., Ph.D., Professor of Chemistry and Physics, The Temple College, Philadelphia, Pa.

Let the origin be the vertex of the cone, O the center of the base of the cone, σ=the length of the dog's path, s=the length of the projection of the dog's path on the plane (x, y) or the base of the cone, r= radius vector of this projection, $a=100$ feet=altitude =radius of base, $m=6$ feet per second, $n=5$ feet per second, $n/m=u$, $x^2+y^2=z^2$ is the equation of the cone. Then $u\sigma=a\theta$, where $\theta=\angle COB$, subtended by the fox's path at the center O.



$$d\sigma=(a/u)d\theta=\sqrt{(dx^2+dy^2+dz^2)}=\sqrt{(ds^2+dz^2)}$$

$$=[r^2+(dr/d\theta)^2+(dz/d\theta)^2]^{\frac{1}{2}}\,d\theta.$$

But $r^2=x^2+y^2=z^2$. $\therefore\ dz=dr$.